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Spin structure in non-forward partons

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Abstract

Renormalisation induces anomalous contributions in light-cone correlation functions. We discuss the role of the axial anomaly and gluon topology in non-forward parton distributions noting that non-forward matrix elements of the gluonic Chern-Simons current K_μ are not gauge-invariant even in perturbation theory. The axial anomaly means that one has to be careful how to interpret information from hard exclusive reactions about the orbital angular-momentum carried by the proton's internal constituents.

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1 Introduction

Non-forward parton distributions [1, 2, 3, 4, 5] have recently been proposed as tools to study hard exclusive reactions: deeply virtual Compton scattering and exclusive meson production [6, 7] in $\gamma^* p$ reactions at large Q^2 (the incident photon virtuality), large s (centre of mass energy) and small t (squared momentum transfer). It has further been proposed that deeply virtual Compton scattering (DVCS) could provide information about quark and gluonic orbital angular-momentum in the proton [2]. It is now well established that the spin structure of the proton is sensitive to the physics of the axial anomaly and dynamical $U_A(1)$ symmetry breaking in QCD – for reviews see [8, 9, 10]. In this paper we discuss the role of these phenomena in non-forward parton distributions and in deeply virtual Compton scattering. The important details are the gauge dependence of non-forward matrix elements of the anomalous Chern Simons current K_+ – even in perturbation theory – and the role of zero modes in dynamical $U_A(1)$ symmetry breaking.

Perturbative QCD factorisation applies to DVCS. In perturbative QCD the amplitudes for deeply virtual Compton scattering and hard exclusive meson production at small t may be written as the integral over the product of non-forward parton distributions and coefficient functions [1-5,11]. The non-forward parton distributions describe the momentum and spin distribution of partons in the target proton and the coefficient functions describe the infra-red safe hard γ^*–parton interaction, which may be calculated in perturbative QCD. Working in light-cone gauge $A_+ = 0$ the spin independent and spin dependent non-forward parton distributions are written as the Fourier transform of light-cone correlation functions: the non-forward matrix elements of "$\bar{q}(0)\gamma_+ q(y)$" and "$\bar{q}(0)\gamma_+\gamma_5 q(y)$" respectively at $y_+ = y_\perp = 0$.

The unpolarised DVCS cross-section receives contributions from both the spin-dependent and spin-independent parts of the DVCS amplitude $\mathcal{A}_{\rm DVCS}$ [2]. Experiments with polarised beams and targets may help to resolve the separate spin-dependent and spin-independent contributions to $\mathcal{A}_{\rm DVCS}$ [12, 13]. Here we assume that both contributions can be extracted from future experimental data and investigate the role of the axial anomaly in the interpretation of the data.

In QCD the bare light-cone correlation functions are ultra-violet divergent requiring renormalisation [14]. This renormalisation induces the anomalous dimensions (Q^2 dependence of the distributions). It also means that the spin dependent (non-forward) parton distribution is sensitive to the axial anomaly [15, 16, 17]. It is important to check whether any non-perturbative physics induced by the anomaly is consistent with the original perturbative QCD factorisation.

The main results of this paper are the following. First, the flavour-singlet part of $\mathcal{A}^{\rm spin}_{\rm DVCS}$ is sensitive to the QCD axial anomaly and does not have a simple canonical interpretation. In full QCD (beyond perturbation theory) whether factorisation provides a complete description of *this contribution* is sensitive to the dynamics of axial U(1) symmetry breaking – the famous $U_A(1)$ problem of the η' mass – which has the potential to induce zero-mode contributions to the flavour-singlet axial-charge $g_A^{(0)}$ and its non-forward generalisation. Second, Ji [2] has shown that information about quark and gluon total angular-momentum in the proton (J_z^q and J_z^g) may be extracted from the forward limit of the spin-independent part of $\mathcal{A}_{\rm DVCS}$. Through the axial anomaly in QCD, some fraction of the intrinsic spin content of the nucleon and of the constituent quark is carried by its quark and gluon partons and some fraction is carried by gluon topology through a zero-mode [8]. The zero-mode term $\mathcal{C}$ is a model independent physical quantity which can, in principle, be measured through νp elastic scattering [18]. The axial anomaly and zero-modes contribute separately to S_z^q and L_z^q but cancels in the sum. Possible zero-modes and scheme dependence mean that values of L_z extracted from future DVCS data have to be labelled with respect to the degrees of freedom and the perturbative QCD scheme dependence used to define S_z^q.

The plan of the paper is as follows. To motivate our discussion of spin in non-forward parton distributions we begin in Section 2 with a brief overview of our present understanding of the proton spin problem from polarised deep inelastic scattering. In Section 3 we give a brief overview of non-forward parton distributions – for excellent reviews see [3, 4]. Section 4 discusses the axial anomaly and Section 5 its role in the spin dependent part of $\mathcal{A}_{\rm DVCS}$. In Section 6 we discuss the role of the anomaly and gluon topology in the interpretation of information about L_z from DVCS. Finally, in Section 7 we make our conclusions.

2 The spin structure of the proton

To motivate our discussion of spin effects in DVCS we first briefly review what is known about the quark and gluonic intrinsic spin structure of the proton from the interpretation of polarised deep inelastic scattering data. We start with the simple sum-rule for the spin $+\frac{1}{2}$ proton

$$\frac{1}{2} \; = \; \frac{1}{2}\Sigma + L_z + S_z^g. \tag{1}$$

Here, $\frac{1}{2}\Sigma$ and S_z^g are the quark and gluonic intrinsic spin contributions to the nucleon's spin and L_z is the orbital contribution. One would like to understand the spin decomposition, Eq.(1), both in terms of the fundamental QCD quarks and gluons and also in terms of the constituent quark quasi-particles of low-energy QCD. In relativistic constituent quark models Σ is given by the flavour-singlet axial-charge $g_A^{(0)}$. The value of $g_A^{(0)}$ extracted from polarised deep inelastic scattering experiments is $g_A^{(0)}|_{\rm pDIS} = 0.2 - 0.35$ [19], roughly half of the value predicted by the constituent quark models. In QCD the interpretation of the individual quantities in Eq.(1) is quite subtle because of the axial anomaly [10], issues of gauge invariance [20, 21] and dynamical $U_A(1)$ symmetry breaking [8, 22, 23, 10]. In QCD the axial anomaly induces various gluonic contributions to $g_A^{(0)}$. Working in light-cone gauge $A_+ = 0$ one finds [24, 25, 26, 27, 8]

$$g_A^{(0)} = \left(\sum_q \Delta q - 3 \frac{\alpha_s}{2\pi} \Delta g \right)_{\rm partons} + \; \mathcal{C} \tag{2}$$

Here $\frac{1}{2}\Delta q$ and Δg are the amount of spin carried by quark and gluon partons in the polarised proton and $\mathcal{C}$ measures the gluon-topological contribution to $g_A^{(0)}$ [18].

In Eq.(2) $\Delta q_{\rm partons}$ is associated with the hard photon scattering on quark and antiquarks with low transverse momentum squared, k_T^2 of the order of typical gluon virtualities in the proton, and $\Delta g_{\rm partons}$ is associated with the hard photon scattering on quarks and antiquarks carrying $k_T^2 \sim Q^2$. Jet studies and semi-inclusive measurements of the nucleon's spin-dependent g_1 structure function may be used to determine Δg and Δq for each flavour [9]. The topology term $\mathcal{C}$ is associated with dynamical axial U(1) symmetry breaking and has support only at Bjorken $x = 0$. It is missed by polarised deep inelastic scattering experiments but could, in principle, be measured in elastic νp scattering [18]. An example how to obtain a finite value for $\mathcal{C}$ is provided by Crewther's theory of quark-instanton interactions [28]. There, any instanton induced suppression of $g_A^{(0)}|_{\rm pDIS}$ (the axial charge carried by partons with finite momentum fraction $x > 0$) is compensated by a shift of axial charge to the zero-mode so that the total axial-charge $g_A^{(0)}$ including $\mathcal{C}$ is conserved.

The quark and gluonic parton decomposition of $g_A^{(0)}$ in Eq.(2) is factorisation scheme dependent. Eq.(2) describes the singlet axial charge in the k_T^2 cut-off parton model and in the AB and JET schemes [29, 30]. In the $\overline{\rm MS}$ scheme the polarised gluon contribution is absorbed into Δq so that $\Delta q_{\overline{\rm MS}} = \Bigl(\Delta q - \frac{\alpha_s}{2\pi}\Delta g\Bigr)_{\rm partons}$ [31].

The QCD $g_A^{(0)}$ is measured by the proton forward matrix element of the flavour singlet axial-vector current.

$$2ms_\mu g_A^{(0)} = \langle P,S|\ J^{GI}_{\mu 5}\ |P,S\rangle_c \tag{3}$$

where $|P,S\rangle$ denotes a proton state with momentum P and spin S and

$$J^{GI}_{\mu 5} = \left[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\right]_{GI} \tag{4}$$

is the gauge-invariantly renormalised singlet axial-vector operator. In each of the "partons", AB and JET schemes Δq and Δg are measured by the forward matrix elements of the partially conserved axial-vector current and gluonic Chern-Simons current contributions to J^{GI}_{+5} between parton states in the light-cone gauge $A_+ = 0$ – see Sections 4 and 5 below. In light-cone gauge the forward matrix elements of the plus components of these gauge dependent currents are invariant under residual gauge degrees of freedom. However, the non-forward matrix elements of these currents are not invariant, even in perturbative QCD. This means that some of the schemes commonly used to analyse polarised deep inelastic data are not gauge invariant when "skewed" or extrapolated away from the forward direction.

In summary, the axial anomaly brings in gauge invariance issues and the possibility of zero-momentum contributions to the individual spin contributions in Eq.(1). How does this physics enter the theory of deeply virtual Compton scattering ?

3 Non-forward parton distributions

Non-forward parton distributions are defined as the Fourier transforms of light-cone correlation functions. Consider an incident proton with mass M and momentum P_μ which gets given momentum $\Delta_\mu = (P' - P)_\mu$ and emerges into the final state with momentum P'_μ. We use $\bar{P}_\mu = \frac{1}{2}(P' + P)_\mu$ to denote the average nucleon momentum. In this paper we follow the notation of Radyushkin [1]. We let x denote the fraction of light-cone momentum $k_+ = xP_+$ carried by a parton in the incident proton and $\zeta = \Delta_+/P_+$ denote the amount of light-cone momentum transfered to the proton.

In perturbative QCD the spin averaged cross-section for deeply virtual Compton scattering receives contributions from both spin independent and spin dependent non-forward parton distributions. The formula suggested by perturbative QCD for the deeply virtual Compton amplitude $M^{IJ} = \mathcal{A}_{\rm DVCS}$ is:

$$\begin{aligned} & M^{IJ}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = \\ & -{\rm e_q}^2 \frac{1}{2\bar{P}^+} \int_{\zeta-1}^{+1} dx \Big[F_q(x,\zeta,t,\mu^2) C_q(x,\zeta,Q^2,\mu^2) + F_g(x,\zeta,t,\mu^2) C_g(x,\zeta,Q^2,\mu^2) \\ & \qquad + \tilde{F}_q(x,\zeta,t,\mu^2)\tilde{C}_q(x,\zeta,Q^2,\mu^2) + \tilde{F}_g(x,\zeta,t,\mu^2)\tilde{C}_g(x,\zeta,Q^2,\mu^2) \Big] + \mathcal{O}\Big(\frac{1}{Q}\Big) \end{aligned} \tag{5}$$

Here (I, J) refer to the polarisation vectors ($\uparrow$ or $\downarrow$) of the initial and final state photons – here both taken as $\uparrow$; F_q and F_g are the spin-independent non-forward quark and gluonic parton distributions and $\tilde{F}_q$ and $\tilde{F}_g$ are the spin-dependent parton distributions. These distributions are defined in Eqs.(7-10) below; μ^2 denotes the renormalisation scale. In Eq.(5)

$$\begin{aligned} C_q^{\uparrow\uparrow} &= C_q^{\downarrow\downarrow} = \left(\frac{1}{x - i\epsilon} + \frac{1}{x - \zeta + i\epsilon}\right) + O(\alpha_s) \\ C_q^{\uparrow\uparrow} &= -C_q^{\downarrow\downarrow} = \left(\frac{1}{x - i\epsilon} - \frac{1}{x - \zeta + i\epsilon}\right) + O(\alpha_s) \end{aligned} \tag{6}$$

are the non-forward quark coefficient functions with $C_q^{\uparrow\downarrow} = C_q^{\downarrow\uparrow} = \tilde{C}_q^{\uparrow\downarrow} = \tilde{C}_q^{\downarrow\uparrow} = 0$. The gluonic coefficients start at order α_s, viz. $C_g, \tilde{C}_g \sim O(\alpha_s)$. In DVCS ζ plays the role of the Bjorken variable $\zeta = x_{\rm Bj}$. Radyushkin [32] has derived conditions on the non-forward distributions which must be satisfied in order that QCD factorisation holds beyond perturbation theory. One of these conditions is that the non-forward parton distributions do not contain any singular behaviour at $x = 0$ or $x = \zeta$. We shall return to this point in Section 5 below.

The spin independent non-forward quark and gluon distributions are:

$$\begin{aligned} F_q(x,\zeta,t) &= \int \frac{dy_-}{8\pi} e^{ixP_+y_-/2} \langle P'|\bar{q}(0)\gamma_+ \, \mathcal{P} e^{i\int_{y_-}^0 dy_- A_+} \, q(y)|P\rangle\Big|_{y_+=y_\perp=0} \\ &= \frac{1}{2\bar{P}_+} \bar{U}(P') \Big[H_q(x,\zeta,t)\, \gamma_+ \, + E_q(x,\zeta,t)\, \frac{i\sigma_{+\alpha}(-\Delta^\alpha)}{2M} \Big] \, U(P) \end{aligned} \tag{7}$$

and

$$\begin{aligned} F_g(x,\zeta,t) &= \frac{1}{xP_+} \int \frac{dy_-}{8\pi} e^{ixP_+y_-/2} \langle P'|G_{+\alpha}(0)\, \mathcal{P} e^{i\int_{y_-}^0 dy_- A_+} \, G^\alpha{}_+(y)|P\rangle\Big|_{y_+=y_\perp=0} \\ &= \frac{1}{2\bar{P}_+^2} \bar{U}(P') \Big[H_g(x,\zeta,t)\, \gamma_+ \, + E_g(x,\zeta,t)\, \frac{i\sigma_{+\alpha}(-\Delta^\alpha)}{2M} \Big] \, U(P) \end{aligned} \tag{8}$$

respectively. In the forward limit $\zeta = 0$ one recovers the spin independent quark and gluon distributions measured in deep inelastic scattering: $(q\pm\bar{q})(x) = \frac{1}{2}(H_q(x,0,0)\mp$

$H_q(-x,0,0))$ and $g(x) = \frac{1}{2}(H_g(x,0,0) - H_g(-x,0,0))$ respectively. The spin dependent distributions are:

$$\begin{aligned}
\tilde{F}_q(x,\zeta,t) &= \int \frac{dy_-}{8\pi} e^{ixP_+ y_-/2} \langle P'|\bar{q}(0)\gamma_+ \ \gamma_5 \mathcal{P} e^{i\int_{y_-}^0 dy_- A_+} \ q(y)|P\rangle\Big|_{y_+=y_\perp=0} \qquad (9)\\
&= \frac{1}{2\bar{P}_+}\bar{U}(P')\left[\tilde{H}_q(x,\zeta,t)\ \gamma_+\gamma_5 \ + \tilde{E}_q(x,\zeta,t)\ \frac{1}{2M}\gamma_5(-\Delta_+)\right] U(P)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{F}_g(x,\zeta,t) &= -\frac{i}{xP_+}\int \frac{dy_-}{8\pi} e^{ixP_+ y_-/2} \langle P'|G_{+\alpha}(0)\ \mathcal{P} e^{i\int_{y_-}^0 dy_- A_+} \ \tilde{G}^\alpha{}_+(y_-)|P\rangle\Big|_{y_+=y_\perp=0} \qquad (10)\\
&= \frac{1}{2\bar{P}_+^2}\bar{U}(P')\left[\tilde{H}_g(x,\zeta,t)\ \gamma_+\gamma_5 \ + \tilde{E}_g(x,\zeta,t)\ \frac{1}{2M}\gamma_5(-\Delta_+)\right] U(P)
\end{aligned}$$

In the forward limit $\zeta = 0$ one recovers the spin dependent quark and gluon distributions measured in deep inelastic scattering: $\Delta(q \pm \bar{q})(x) = \frac{1}{2}(\tilde{H}_q(x,0,0) \pm \tilde{H}_q(-x,0,0))$ and $\Delta g(x) = \frac{1}{2}(\tilde{H}_g(x,0,0) + \tilde{H}_g(-x,0,0))$ respectively. The isotriplet combination $(\tilde{E}_u - \tilde{E}_d)$ contains the pion pole and the flavour-singlet combination $(\tilde{E}_u + \tilde{E}_d + \tilde{E}_s)$ is sensitive to the axial U(1) problem [28, 33] through the η' pole in the pseudoscalar form-factor.

We now focus on the spin-dependent non-forward quark distribution $\tilde{F}_q$ to discuss the construction and renormalisation of these distributions. The parton model and the light-cone correlation functions are normally formulated in the light-cone gauge $A_+ = 0$. Here the path-ordered exponential becomes a trivial unity factor and is dropped from the formalism:

$$\langle P'|\bar{q}(0)\ \gamma_+\ \gamma_5\ \mathcal{P} e^{i\int_{y_-}^0 dy_- A_+}\ q(y)|P\rangle \ \mapsto\ \langle P'|\bar{q}(0)\ \gamma_+\ \gamma_5\ q(y)|P\rangle \qquad (11)$$

In semi-classical QCD, before we come to discuss renormalisation and anomalies, the gluonic degrees of freedom have dropped out along with the path-ordered exponential – hence the terminology "quark distribution". Before we consider subtleties associated with anomaly theory – see Section 4 below – the non-forward parton distributions have the following simple interpretation [4, 5]. Expanding out the quark and gluon field operators in $q(y)$ and $G_{\mu\nu}(y)$ one finds the following. For $\zeta < x < 1$ we have the situation where one removes a quark carrying light-cone momentum $k_+ = xP_+$ and transverse momentum $\vec{k}_\perp$ from the initial state proton and re-inserts it into the final state proton with the same chirality but with light-cone momentum fraction $x - \zeta$ and transverse momentum $\vec{k}_\perp - \vec{\Delta}_\perp$. For $\zeta - 1 < x < 0$ one finds the situation for removing an antiquark with momentum fraction $\zeta - x$ and re-inserting it with momentum fraction $-x$. For $0 < x < \zeta$ the photons scatter off a virtual quark-antiquark pair in the initial proton wavefunction: the quark of the pair has light-cone momentum fraction x and transverse momentum $\vec{k}_\perp$, whereas the antiquark has light-cone momentum fraction $\zeta - x$ and transverse momentum $\vec{\Delta}_\perp - \vec{k}_\perp$. The third region $0 < x < \zeta$ is not present in deep inelastic scattering where $\zeta = 0$. The points $x = 0$ and $x = \zeta$ correspond to zero-momentum modes. The flavour-singlet spin-dependent parton distributions at these two points are sensitive to the role of zero-modes in dynamical $U_A(1)$ symmetry breaking [28, 33], which have the potential to generate new non-perturbative contributions to the spin-dependent part of $\mathcal{A}_{DVCS}$ beyond those appearing in the factorisation formula (5). The $J = 0$ Regge fixed pole [34] in the spin-independent part of $\mathcal{A}_{DVCS}$ which generates a constant real term in $\mathcal{A}_{\rm DVCS}$ is manifest in Eqs.(5,6) through the $\zeta \to 0$ limit of C_q.

The x-moments, $\int_{\zeta-1}^{+1} dx\, x^n$, of the non-forward parton distributions are evaluated as follows. First one writes x^n as a derivative (in y_-) acting on $e^{ixP_+y_-/2}$. Integrating by parts (with respect to x) over the exponential yields a Dirac delta-function $\delta(y_-)$. The y_- integral then projects out the non-forward matrix elements of local operators. One finds

$$\begin{aligned}
&\int_{\zeta-1}^{1} dx\ x^n\ \tilde{F}_q(x,\zeta,t) \qquad (12)\\
&= \frac{1}{2}\Big(\frac{2}{P_+}\Big)^n \langle P'|\bar{q}(0)\gamma_+\ \gamma_5\ (i\partial_+)^n\ q(0)|P\rangle\Big|_{y_+=y_\perp=0}\\
&= \frac{1}{2\bar{P}_+}\bar{U}(P')\Big[\int_{\zeta-1}^{+1} dx\ x^n\ \tilde{H}_q(x,\zeta,t)\ \gamma_+\gamma_5\ + \int_{\zeta-1}^{+1} dx\ x^n\ \tilde{E}_q(x,\zeta,t)\ \frac{1}{2M}\gamma_5(-\Delta_+)\Big]\ U(P)
\end{aligned}$$

A caveat is due here: renormalisation means that we have to be careful not to simply set $y_- = 0$ in the point-split operator to obtain the local operator. In QCD the bare light-cone correlation functions are ultra-violet divergent requiring renormalisation [14]. This renormalisation means that spin independent (non-forward) parton distribution $\tilde{F}_q$ is sensitive to the axial anomaly.

We now discuss the effect of the anomaly in non-forward parton distributions.

4 Renormalisation and anomalies

Going beyond the semi-classical approximation to QCD we have to take into account that the renormalised composite operator

$$\Big[\ \bar{q}\gamma_\mu\gamma_5 q\ \Big]^R_{\mu^2}(0)\ \neq\ \bar{q}(0)\gamma_\mu\gamma_5\ \ \text{multiplied by}\ \ q(0) \qquad (13)$$

This point is especially important when we evaluate the integral over y_- of the point-split matrix element with $\delta(y_-)$. Evaluating the moments of $\tilde{F}_q$ is non-trivial because the point-split operator is highly singular in the limit that the point splitting is taken to zero. In full QCD it is necessary to work with renormalised composite operators instead of their semi-classical prototypes. To see this explicitly consider Schwinger's derivation [17, 35] of the axial anomaly using point split regularisation. For $(z' - z'') \to 0$, one finds that the vacuum to vacuum matrix element of the point-split operator in an external gluon field is:

$$\langle\text{vac}|\ \Big[\overline{q}(z')\gamma_+\gamma_5 q(z'')\Big]\ |\text{vac}\rangle \simeq \frac{ig}{8\pi^2}\ \tilde{G}_{+\nu}\ \frac{(z'-z'')^\nu}{(z'-z'')^2} \qquad (14)$$

where the gluonic term arises from pinching a gluonic insertion between z' and z''. Going to the light-cone ($z_T = 0$, $z_+ \to 0$) the factor

$$\frac{(z'-z'')_+}{(z'-z'')^2}\quad \mapsto \quad \frac{1}{(z'-z'')_-} \qquad (15)$$

which diverges when $z' \to z''$. One clearly has to be careful and not ensure that the theory and its interpretation does not assume equality of both sides in Eq.(13).

The problem is resolved [14] if, working in light-cone gauge, we define

$$\langle P', S'| \Big[\overline{q}(0)\gamma_+\gamma_5 q(y_-)\Big]^R_{\mu^2} |P,S\rangle_c \equiv \sum_n \frac{(iy_-)^n}{n!} \langle P', S'| \Big[\overline{q}\gamma_+\gamma_5 (iD_+)^n q\Big]^R_{\mu^2}(0)|P,S\rangle_c \tag{16}$$

and

$$\langle P', S'|\mathrm{Tr}\Big[G_{+\nu}(0)\tilde{G}^{\nu}{}_{+}(y_-)\Big]^R_{\mu^2} |P,S\rangle_c \equiv \sum_n \frac{(iy_-)^n}{n!} \langle P', S'|\mathrm{Tr}\Big[G_{+\nu}(iD_+)^n \tilde{G}^{\nu}_{+}\Big]^R_{\mu^2}(0)|P,S\rangle_c \tag{17}$$

That is, we treat the non-local operators in Eqs.(7-10) as a series expansion in terms of renormalised composite local operators in the operator product expansion. The superscript R denotes the renormalisation prescription and μ^2 denotes the renormalisation scale. The composite operators $\Big[\overline{q}\gamma_+\gamma_5 (iD_+)^{2n} q\Big](0)$ and $\mathrm{Tr}\Big[G_{+\nu}(iD_+)^{2n}\tilde{G}^{\nu}_{+}\Big](0)$ mix under renormalisation. The axial vector and higher-spin axial tensor operators are sensitive to the axial anomaly.

4.1 The axial anomaly

The gauge-invariantly renormalised flavour singlet axial-vector current

$$J^{GI}_{\mu 5} = \Big[\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s\Big]^{GI}_{\mu^2} \tag{18}$$

satisfies the anomalous divergence equation [15, 16]

$$\partial^\mu J^{GI}_{\mu 5} = 2f\partial^\mu K_\mu + \sum_{i=1}^{f} 2im_i \bar{q}_i\gamma_5 q_i \tag{19}$$

Here

$$K_\mu = \frac{g^2}{16\pi^2}\epsilon_{\mu\nu\rho\sigma}\Big[A^\nu_a\Big(\partial^\rho A^\sigma_a - \frac{1}{3}g f_{abc}A^\rho_b A^\sigma_c\Big)\Big] \tag{20}$$

is a renormalised version of the gluonic Chern-Simons current and the number of light flavours f is 3. Eq.(20) allows us to write

$$J^{GI}_{\mu 5} = J^{\mathrm{con}}_{\mu 5} + 2fK_\mu \tag{21}$$

where $J^{\mathrm{con}}_{\mu 5}$ and K_μ satisfy the divergence equations

$$\partial^\mu J^{\mathrm{con}}_{\mu 5} = \sum_{i=1}^{f} 2im_i \bar{q}_i\gamma_5 q_i \tag{22}$$

and

$$\partial^\mu K_\mu = \frac{g^2}{8\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}. \tag{23}$$

Here $\frac{g^2}{8\pi^2}G_{\mu\nu}\tilde{G}^{\mu\nu}$ is the topological charge density. The partially conserved current is scale invariant and the scale dependence of $J^{GI}_{\mu5}$ is carried entirely by K_μ. When we make a gauge transformation U the gluon field transforms as

$$A_\mu \to UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \tag{24}$$

and the operator K_μ transforms as

$$K_\mu \to K_\mu + i\frac{g}{16\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu\Big(U^\dagger\partial^\alpha UA^\beta\Big) + \frac{1}{96\pi^2}\epsilon_{\mu\nu\alpha\beta}\Big[(U^\dagger\partial^\nu U)(U^\dagger\partial^\alpha U)(U^\dagger\partial^\beta U)\Big]. \tag{25}$$

Gauge transformations shuffle a scale invariant operator quantity between the two operators $J^{\rm con}_{\mu5}$ and K_μ whilst keeping $J^{GI}_{\mu5}$ invariant.

The non-abelian three-gluon part of K_+ vanishes in $A_+ = 0$ gauge and the forward matrix elements of K_+ are invariant under residual gauge degrees of freedom in this gauge. Furthermore the forward matrix elements of K_+ measure the amount of spin carried by gluonic partons in the target [21]. This leads ultimately to the "partons", AB and JET scheme decompositions of $g_A^{(0)}$ in Section 2. As soon as we go away from the forward direction matrix elements of K_+ will pick up a gauge-dependent contribution, which must decouple from any physical observable.

4.2 The axial anomaly and higher moments

The anomaly is also present in the $C = +1$ higher spin axial tensors [36]. In general, for a given choice of renormalisation prescription R, the renormalised axial tensor operator differs from the gauge invariant operator by a multiple of a gauge-dependent, gluonic counterterm $K_{\mu\mu_1...\mu_{2n}}$, viz.

$$\begin{aligned}\Big[\overline{q}\gamma_\mu\gamma_5 iD_{\mu_1}...iD_{\mu_{2n}}q\Big]^R_{Q^2}(0) \quad &= \\ \Big[\overline{q}\gamma_\mu\gamma_5 iD_{\mu_1}...iD_{\mu_{2n}}q\Big]^{GI}_{Q^2}(0) \quad + \quad \lambda^{(K)}_{R,n}\ \Big[K_{\mu\mu_1...\mu_{2n}}\Big]_{Q^2}&(0) + \lambda^{(G)}_{R,n}\ \Big[G_{\mu\alpha}iD_{\mu_1}...\tilde{G}^\alpha{}_{\mu_{2n}}\Big]^{(GI)}_{Q^2}(0)\end{aligned} \tag{26}$$

Shifting between possible renormalisation schemes, we pick up contributions from counterterms involving the gauge-invariant gluonic operators $G_{\mu\alpha}iD_{\mu_1}...iD_{\mu_{2n-1}}\tilde{G}^\alpha_{\mu_{2n}}$ and also the gauge-dependent $K_{\mu\mu_1...\mu_{2n}}$. The coefficients $\lambda^{(K)}_{R,n}$ and $\lambda^{(G)}_{R,n}$ are fixed by the choice of renormalisation prescription. In $A_+ = 0$ gauge the two-gluon part of $K_{\mu\mu_1...\mu_{2n}}$ reads

$$K_{+...+(2n+1)} = \frac{\alpha_s}{\pi}\lambda^{(K)}_n\ \epsilon_{+\lambda\alpha\beta}\ A^\alpha\partial^\lambda(i\partial_+)^{2n}A^\beta \tag{27}$$

It is not easy to derive the $K_{\mu\mu_1...\mu_{2n}}$ beyond the two gluon term. Unlike the topological charge density $\frac{\alpha_s}{4\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$ the higher spin operators $\frac{\alpha_s}{4\pi}G_{\mu\nu}iD_{\mu_1}...iD_{\mu_{2n}}\tilde{G}^{\mu\nu}$ are not topological invariants for $n \geq 1$. It follows that they are not total derivatives and, therefore, one cannot use a divergence equation alone to fix the non-abelian part of $K_{\mu\mu_1...\mu_{2n}}$. There is no equation $\partial^{\mu_j}\mathcal{S}\mathcal{K}_{\mu\mu_1...\mu_{2n}} = \frac{\alpha_s}{4\pi}G_{\mu\nu}iD_{\mu_1}...iD_{\mu_{2n}}\tilde{G}^{\mu\nu}$ for $n \geq 1$.

5 The axial anomaly in $\tilde{F}_q$

The first observation to make is that the non-forward matrix elements of K_+ (and $K_{+...+(2n+1)}$) are gauge dependent in $A_+ = 0$ gauge, even in perturbation theory. This means that these operators must decouple from any factorisation scheme or "generalised operator product expansion" [2] for hard exclusive reactions like deeply virtual Compton scattering. This is in contrast to the situation in deep inelastic scattering where the forward matrix elements of K_+ are invariant under residual gauge degrees of freedom in $A_+ = 0$ gauge.

Since there is no gauge-invariant local gluonic operator with quantum numbers $J^{PC} = 1^{++}$ it follows that the first moment of the spin-dependent *non-forward* gluonic coefficient must vanish. This holds true in the non-forward generalisation of the $\overline{\mathrm{MS}}$ factorisation scheme for handling the infra-red mass singularities. [1] However, there is no gauge-invariant non-forward generalisation of the popular JET and AB schemes used to describe polarised deep inelastic data. In these schemes there is an explicit gluonic contribution to the first moment of g_1 associated with the invariant contribution of K_+ (in $A_+ = 0$ gauge) to the forward matrix element of J^{GI}_{+5} induced by a non-vanishing first moment of the spin-dependent gluonic coefficient.

The gauge dependence of the non-forward matrix elements of K_+ also means that one has to be careful with the interpretation of the spin-dependent non-forward parton distributions in terms of amplitudes to extract and then re-insert a (well defined) parton with a given momentum. Consider the non-forward spin-dependent quark distribution defined first using gauge invariant renormalisation and second via the partially conserved axial vector current. In the first case, gluonic information is intrinsically built into the definition of the "polarised quark" via the axial anomaly. In the second, the notion of "polarised quark parton" is no longer gauge invariant. Taken together, this means that one has to be careful how far one carries through the semi-classical interpretation of parton distributions. The gluonic information built into the flavour-singlet part of $\tilde{F}_q$ is manifest through the massive η' and absence of any (nearly-)massless pseudo-Goldstone axial U(1) pole in $\tilde{E}_q$.

The zero-modes associated with dynamical $U_A(1)$ symmetry breaking mean that the perturbative QCD formula (5) for deeply virtual Compton scattering may not necessarily exhaust the total cross-section. For example, in Crewther's theory of quark-instanton interactions one would expect $\delta(x)$ and $\delta(x - \zeta)$ contributions in the flavour-singlet part of $\tilde{H}_q(x, \zeta, t)$ associated the transfer of incident axial-charge carried by quarks and antiquarks respectively from partons with finite momentum to zero-modes. Such zero-mode contributions are not easily reconcilable with the perturbative QCD factorisation expression (5), in particular the singularities in the leading-twist coefficient function $\tilde{C}_q$ in Eq.(6). This problem is not relevant to isovector pion or rho production [7] which is described just in terms of the isovector non-forward distributions. Any flavour-singlet zero-modes will not contribute to these processes.

[1] See eg. the calculations of Mankiewicz et al. [37] and Ji and Osborne [2]; in the notation of Mankiewicz et al. [37] $\frac{\partial}{\partial u} C_1^{A,g} \Big|_{u=0} = 0$.

6 Orbital angular momentum

A sum-rule [2, 39] relates the form-factors appearing in F_q and F_g in the spin-independent part of $\mathcal{A}_{\rm DVCS}$ to the quark and gluonic total angular-momentum contributions to the spin of the proton. The second moments of F_q and F_g project out the non-forward matrix elements of the QCD energy momentum tensor:

$$\begin{aligned}\langle P'|\, T^{\mu\nu}_{q,g}\, |P\rangle = \bar{U}(P')\Big[A_{q,g}(t)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(t)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M \\ +C_{q,g}(t)\Delta^{(\mu}\Delta^{\nu)}/M + \mathcal{O}(\Delta^3)\Big]U(P)\end{aligned} \tag{28}$$

There are no massless bosons which couple to $T^{\mu\nu}$ in QCD, which is associated with the fact that Poincare invariance is not spontaneously broken in QCD (with corresponding Goldstone bosons). This means that the expansion in Δ and the forward limit of the form-factors in Eq.(28) are well-defined: there are no $\Delta_\mu\Delta_\nu/\Delta^2$ terms. The current associated with Lorentz transformations is

$$M_{\mu\nu\lambda} = z_\nu T_{\mu\lambda} - z_\lambda T_{\mu\nu} \tag{29}$$

The total angular momentum operator is related to the energy-momentum tensor by

$$J^z_{q,g} = \langle P', \frac{1}{2}|\ \int d^3z\ (\vec{z} \text{ x } \vec{T}_{q,g})^z\ |P, \frac{1}{2}\rangle \tag{30}$$

Substituting Eq.(28) into Eq.(30) and taking the forward limit $\Delta \to 0$ one obtains

$$J^z_{(q,g)} = \frac{1}{2}\Big[A_{q,g}(0) + B_{q,g}(0)\Big] \tag{31}$$

This result is Ji's sum-rule [2] relating total angular momentum to the form-factors measured appearing in Eqs.(7) and (8).

If one can extract $J_{q,g}$ from the forward limit of $A(t)$ and $B(t)$ in hard exclusive processes, then subtracting S_z from polarised deep inelastic scattering would give information about the orbital angular momentum L_z. Whereas the intrinsic spin S^q_z is sensitive to the axial anomaly, the total angular momentum $J_z^{(q+g)}$ is not because it is measured by a conserved current. Theoretical studies [26, 38] show that J^q_z and J^g_z are each anomaly free in perturbative QCD meaning that the axial anomaly cancels between S^q_z and L^q_z in perturbation theory [26, 38, 39].

This result generalises beyond perturbation theory [39] with the added consequence that there is no zero-mode contribution to J_q and J_g. To see this, first consider the crossing symmetry in x of the spin-independent quark distributions. The second moment of the charge parity plus distribution $(q+\bar{q})(x)$ projects out the nucleon matrix element of the energy-momentum tensor $T_{\mu\nu}$. Any zero mode contribution to the right-hand side of the energy-momentum sum-rule would be associated with a $\delta'(x)$ term in $(q+\bar{q})(x)$. Less singular $\delta(x)$ terms may be induced in the spin-dependent distributions by quark instanton interactions. I know of no model which predicts a stronger singularity like $\delta'(x)$. Phenomenologically, such a term in the spin-independent structure function F_2 would lead to a violation of the energy-momentum sum-rule for partons, which is not observed. Since there is no zero-mode contribution to J_z it follows that any zero-mode contribution to $g_A^{(0)}$ is compensated by a second zero-mode with equal magnitude but opposite sign in the orbital angular momentum L_z^q. This has the practical consequence that one has to be careful how one interprets any determination of L_z^q from DVCS through the sum-rule (31). The orbital angular momentum carried by constituent quarks and by "current quark" partons are not necessarily the same, and is distinguished by the topological term $\mathcal{C}$ measurable in νp elastic scattering. What happens to spin and orbital angular momentum in the transition from current to constituent quarks is intimately related to the dynamics of axial U(1) symmetry breaking.

Finally, we note that in perturbative QCD one also has to be careful to quote any value of "L_z^q" with respect to the factorisation scheme used to extract "S_z^q" from polarised deep inelastic data. For example, $\Delta q_{\overline{\rm MS}} = (\Delta q - \frac{\alpha_s}{2\pi}\Delta g)_{\rm AB,\ JET}$ – see also Shore [40].

7 Conclusions

In summary, the axial anomaly is manifest in deeply virtual Compton scattering directly through the flavour-singlet spin-dependent part of the scattering amplitude $\mathcal{A}_{\rm DVCS}$ through the non-forward parton distribution $(\tilde{F}_u + \tilde{F}_d + \tilde{F}_s)$ and indirectly through the interpretation of information about quark orbital angular-momentum extracted from the spin-independent part of $\mathcal{A}_{\rm DVCS}$.

Whether QCD factorisation provides a complete description of the flavour-singlet, spin-dependent part of $\mathcal{A}_{\rm DVCS}$ depends on possible zero-mode contributions to $(\tilde{F}_u + \tilde{F}_d + \tilde{F}_s)$ which may be generated by the dynamics of axial U(1) symmetry breaking. In the absence of such zero-mode contributions perturbative QCD factorisation works for this term provided one chooses a gauge-invariant factorisation procedure such as minimal subtraction. However the resultant spin-dependent non-forward distribution will not have a simple canonical interpretation in terms of extracting and re-inserting a quark or antiquark in the target because gluonic information is intrinsic to $(\tilde{F}_u + \tilde{F}_d + \tilde{F}_s)$ through the anomaly. This gluonic information is manifest in the large-mass η' pole in $(\tilde{E}_u + \tilde{E}_d + \tilde{E}_s)$ associated with the flavour-singlet pseudoscalar form-factor.

QCD anomaly effects cancel between the intrinsic spin and orbital angular-momentum contributions to the total quark angular-momentum in the proton. This extends to possible zero-mode contributions associated with dynamical $U_A(1)$ symmetry breaking. In QCD some fraction of the intrinsic spin carried by low-energy constituent quark quasi-particles may be carried by a zero-mode in addition to the partonic contributions from finite Bjorken x. This zero-mode is compensated by a zero-mode with equal magnitude and opposite sign in the orbital angular-momentum so that the total quark angular-momentum is independent of the details of the current to constituent quark transition through $\mathcal{C}$ whereas the separate intrinsic and orbital contributions are not. Finally, any value of the orbital angular-momentum extracted from future DVCS experiments should also be quoted with respect to the perturbative QCD factorisation scheme used to extract the partonic intrinsic spin from polarised deep inelastic data.

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